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Original Article

Application of Pontryagin's Minimum Principle in the Artificial Neural Network to Reduce the COVID-19 Pandemic Effects

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Abstract

Introduction: In this study, we analyze the optimal intervention strategies that lead to reducing the effects of the COVID-19 pandemic by artificial neural networks (ANNs). Our aim is to investigate the effects of optimal control strategies, such as the implementation of government intervention, testing, and vaccination policies during outbreaks.

Methods: We utilized a controlled SIDAREV model to study the progression of the COVID-19 pandemic. Using Pontryagin's minimum principle (PMP) for the SIDAREV model, we defined an unconstrained minimization problem. Applying the Hamiltonian conditions, we approximated the obtained ordinary differential equations (ODE) using ANNs. We utilized the multilayer perceptron (MLP) to obtain the approximate solution of the states and co-states functions.

Results: We observed the effects of optimal control strategies, and to show the efficiency of the proposed method, we compared it with the Runge-Kutta method through some examples.

Conclusion: Using a mathematical model that simulates the behavior of the Covid-19 disease, we can examine the effects of controllers such as government interventions, tests and vaccinations with the neural network method. The results show that this method is useful in solving the problem of optimal control of infectious diseases.

Keywords: Optimal control, Pontryagin's minimum principle, Artificial neural network, SIDAREV model, COVID-19

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Introduction

At the end of 2019, a new coronavirus was reported in Wuhan, China (1). This virus, which spread rapidly in China and other parts of the world, is known as severe acute respiratory syndrome coronavirus 2 (SARS-CoV-2) (2). On March 17, 2020, the World Health Organization (WHO) officially declared the coronavirus as a pandemic. Due to the current situation, in addition to facing a novel human tragedy, the world was faced with the fear of economic failure. More than 6 million deaths and 500 million infectious cases were reported by the WHO in the time of writing this article. Headache, difficulty in breathing, fever, loss of smell, cough, and loss of taste can be included among the symptoms of this deadly disease. Despite strict measures to control and prevent the infection, types of this disease, such as alpha, beta, delta, and now the Omicron type, among the effects of which is an increase in the number of infected people, followed by an increase in the death rate appeared all over the world (3). Governments around the world implemented extensive non-pharmacological interventions to combat the effects of this virus, such as quarantine policies and social distancing, banning public events, personal hygiene and cough etiquette, closure of schools, wearing face masks, and self-isolation (4). At first glance, such interventions reduce the spread of the virus and the infection rate of the disease (5), but in the long term, it imposes severe negative effects on the economy, which can lead to the most severe global recession in more than 40 years and the loss of more than 5% of the gross domestic product in developed countries (6). Therefore, the implementation of policies such as quarantine and social distancing may be effective in containing the virus, but it is economically costly for governments and naturally turns such decisions into a multi-objective problem. With the advent of the Covid-19 disease and its types, mathematical models were used to formulate effective policies to reduce their impact and describe the dynamic evolution of the pandemics (7). The widely used susceptible-infected-recovered (SIR) model is analyzed in (8). For a comprehensive review of epidemiology models, the reader is referred to (9). In these models, it is possible to study the progression of various diseases over time, observe the dependence on model parameters, and facilitate the description of their asymptotic behavior. Recently, researchers have proposed different approaches to model the progression of the COVID-19 outbreak. For instance, in (10), a common approach is to use different extensions of the SIR model. In (11), a time-varying susceptible-infected-recovered-deceased (SIRD) model has been proposed. Also, in (12), Giordano et al. used a more involved compartmental model, offering larger modeling flexibility compared to simpler models. Rossa, in (13), established the regional heterogeneity of the pandemic by a developed model.

In general, our goal in this article is to analyze a model that, in addition to reducing the socio-economic cost, can reduce the number of people who die due to this disease to the lowest possible amount. Therefore, we should design the optimal control strategy related to measures such as government intervention, testing, and vaccination policies during a disease outbreak. To achieve this strategy, we defined the SIDAREV (susceptible, infected undetected, infected detected, acutely symptomatic, recovered, extinct, vaccinated) model that takes into account the effect of government intervention policies. In this model, it is possible to integrate features such as the impact of existing healthcare capacity, testing and vaccination rates.

In this study, to investigate the problem of forming a practical intervention strategy and the efficiency of the government, we used the theory of optimal control, which limits the number of deaths caused by the COVID-19 pandemic. Research has already been conducted on optimal control in epidemics (especially due to the COVID-19 pandemic). In (14), an optimal control strategy was investigated in which the number of deaths and the costs related to implementing the control strategy was minimized. In (15), an optimal control analysis was performed, which showed that optimal preventive strategies such as public health education, personal protective measures, and treatment of hospitalized cases effectively reduced the number of COVID-19 deaths. In (16), two optimal control policies were analyzed. The first is the open-loop optimal control policy, in which the number of fatalities can be decreased significantly under the assumption of exact model knowledge. They stated that this was not a realistic scenario in the real world since it should deal with uncertain data and model mismatch. Therefore, they designed a feedback strategy that updated the policy weekly using model predictive control. Also, they found that this feedback control was robust and necessary for reliably handling an outbreak. Other studies for optimal control can be found in (17). The model used was a modified version of the existing SIDARE model used in (18). In this study, a vaccination compartment was added to the model; therefore, this model is called SIDAREV. In (19), the vaccination compartment was omitted and linked the vaccination parameter ψ directly to the recovered compartment. ψ described the rate at which susceptible individuals got vaccinated.

Now, we are looking for a method to solve the model resulting from the optimal strategy problem. Many researchers have studied optimal control theory to investigate measures to control and reduce the effects of the disease (20-22). In this work, we are going to use ANNs to analyze the optimal control strategies designed on the SIDAREV model. ANNs are one of the effective methods that have been used in recent decades to solve various nonlinear problems, and their results can be compared with other problems using mathematical algorithms. In (23), ANNs were used to solve ODEs and PDEs for boundary and initial value problems. Based on a reinforcement learning scheme, Vrabie et al. in (24) solved continuous-time direct adaptive optimal control for partially unknown nonlinear systems. The theory and applications, algorithms, modeling, design, and mathematics of neural networks can be found in many sources (25), particularly the numerical solution of ordinary and partial differential equations (26, 27), mathematical programming (28, 29) and optimal control problems (30, 31). In (32), the fuzzy neural networks (FNNs) algorithm was utilized for detecting cardiovascular diseases. Using various types of ANN, Acar et al. applied the backpropagation (BP) algorithm for forecasting diabetes mellitus (33). In (34), Afshar et al. used the Levenberg-Marquardt learning algorithm for the recognition and prediction of leukemia. Khemphila et al. applied ANNs for heart disease classification. MLP with a BP learning algorithm was employed for neonatal disease diagnosis in (35). Heydari Dastjerdi et al. investigated the SEIR epidemic model related to infectious diseases by using artificial neural network (36). This study aimed to use the ability of ANNs to approximate the states and co-states functions of the SIDAREV pandemic model.

Model Description

The SIDAREV model has seven components, and we express the relationship between these components. It is assumed that β denotes the infection rate for susceptible individuals, ν denotes the rate of detection of infected individuals based on the level of testing, $\gamma_i, \gamma_d, \gamma_a$ describe the recovery rate for infected undetected, infected detected, and acutely symptomatic (threatened) individuals, respectively, ξ_i and ξ_d denote the rate when infected individuals become acutely symptomatic (threatened) and, ψ denotes the rate at which susceptible individuals get vaccinated. We aimed to examine the effect of three controllers on the SIDAREV model, so we will introduce these controllers. Three control inputs (u_1, u_2, and u_3) indicating the strength of the government interventions, strength of the testing policy, and strength of the vaccination policy, respectively, were added to the SIDAREV model. In (18), the impact of healthcare capacity on the mortality rate was included; once the healthcare capacity exceedes, the mortality rate will increase. On the other hand, regular care can no longer take place at this time; as a result, many more individuals die. This change can be modeled as follows:

$$\bar{\mu}(A) = \begin{cases} \mu A, & A \le \bar{h}, \\ \mu\bar{h} + \hat{\mu}(A - \bar{h}), & A > \bar{h}, \end{cases}$$

where the function $\bar{\mu}: R \to R$ describes the mortality of the acutely symptomatic population. Furthermore, $\hat{\mu}$ is five times higher than the current mortality rate, i.e. $\hat{\mu} = 5\mu$, and $\bar{h}$ indicates the hospital capacity. Given all the stated assumptions, the dynamics of the SIDAREV model including the change in the mortality rate and the controllers can be constructed as follows:

$$\begin{aligned}
\dot{S} &= -\beta SI(1-u_1) - \psi S u_3, \\
\dot{I} &= \beta SI(1-u_1) - \gamma_i I - \xi_i I - \nu I u_2, \\
\dot{D} &= \nu I u_2 - \gamma_d D - \xi_d D, \\
\dot{A} &= \xi_i I + \xi_d D - \gamma_a A - \bar{\mu}(A), \\
\dot{R} &= \gamma_i I + \gamma_d D + \gamma_a A, \\
\dot{E} &= \bar{\mu}(A), \\
\dot{V} &= \psi S u_3.
\end{aligned} \qquad (1)$$

A schematic diagram of the SIDAREV model is shown in Figure 1.

Note that all model parameters are constant and non-negative. The SIDAREV model is based on the following assumptions:

- The considered population is constant, i.e., no births or deaths not attributed to a particular disease outbreak are taken into account.
- Infected individuals that are detected are assumed to be quarantined immediately, so that they do not contribute to new infections.
- Infected individuals become first acutely symptomatic before they die.
- Acutely symptomatic individuals require hospitalization since they are considered threatened with decease.
- Only susceptible individuals are vaccinated.
- Vaccinated individuals are immune to the disease and, thus, cannot become susceptible anymore.

Our purpose in this model is to reduce the number of threatened individuals, the use of government intervention, testing, and vaccination and finally, the number of deceased individuals in the final time T. Therefore, we define the objective functional as follows.

$$\text{Min } J(A, E, u_1, u_2, u_3) = \int_0^T \left(\frac{c_1}{2} A(t)^2 + \frac{b_1}{4} u_1(t)^2 + \frac{b_2}{4} u_2(t)^2 + \frac{b_3}{4} u_3(t)^2 \right) dt + c_2 E(T),$$

where c_1 is the positive weight coefficient to obtain balance in the optimization function, and b_1, b_2, and b_3 measure the relative cost of the optimal control input and can be adjusted as desired. As with the first term, c_2 provides a positive weight factor to balance the optimization function. The reason for the second power of the variables is the ease of solving the model.

It should be noted that there are limitations for controllers. It is assumed that the number of infections can be decreased at its maximum value by 80%, so the maximum value that u_1 can take is 0.8. For control of input u_2, when the control input equals 0, the detection rate of infected individuals is 0%. When the control input equals 1, the detection rate of infected individuals is 100%; thus, the maximum value that u_2

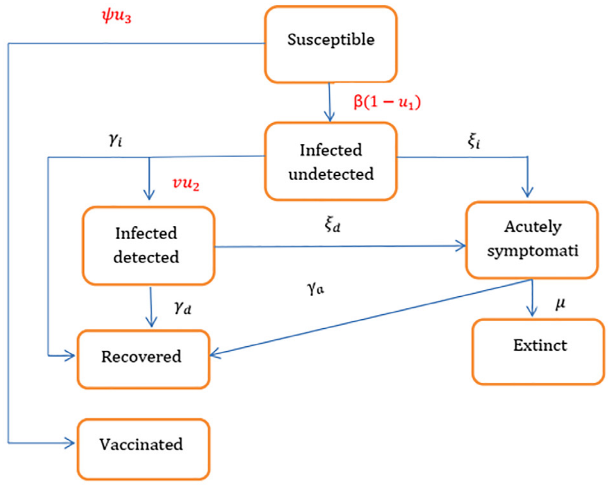

Figure 1: Schematic diagram of the SIDAREV model

can take is 1. Finally, the maximum value that u_3 can take is 1. When no control input is applied, the rate of vaccination of susceptible individuals is 0%. When the maximal control input is used, the vaccination rate of susceptible individuals is 100%. Given the objective functional and control inputs, the optimal control problem can be described as follows:

$$\text{Min } J(A, E, u_1, u_2, u_3) = \int_0^T \left(\frac{c_1}{2}A(t)^2 + \frac{b_1}{4}u_1(t)^2 + \frac{b_2}{4}u_2(t)^2 + \frac{b_3}{4}u_3(t)^2 \right) dt + c_2 E(T),$$

$$\begin{aligned}
&\dot{S}(t) = -\beta S(t)I(t)\big(1 - u_1(t)\big) - \psi A(t)u_3(t),\\
&\dot{I}(t) = \beta S(t)I(t)\big(1 - u_1(t)\big) - \gamma_i I(t) - \xi_i I(t) - \nu I(t)u_2(t),\\
&\dot{D}(t) = \nu I(t)u_2(t) - \gamma_d D(t) - \xi_d D(t),\\
&\dot{A}(t) = \xi_i I(t) + \xi_d D(t) - \gamma_a A(t) - \bar{\mu}\big(A(t)\big),\\
&\dot{R}(t) = \gamma_i I(t) + \gamma_d D(t) + \gamma_a A(t),\\
&\dot{R}(t) = \gamma_i I(t) + \gamma_d D(t) + \gamma_a A(t),\\
&\dot{E}(t) = \bar{\mu}\big(A(t)\big),\\
&\dot{V}(t) = \psi S(t)u_3(t),\\
&0 \le u_1(t) \le 0.8,\\
&0 \le u_2(t) \le 1,\\
&0 \le u_3(t) \le 1,
\end{aligned} \tag{2}$$

$$S(0) = S_0, I(0) = I_0, D(0) = D_0, A(0) = A_0, R(0) = R_0, E(0) = E_0, V(0) = V_0,$$

where $S(t)$, $I(t)$, $D(t)$, $A(t)$, $R(t)$, $E(t)$, and $V(t)$ represent the number of susceptible, infected undetected, infected detected, acutely symptomatic, recovered, extinct, and vaccinated individuals, respectively at the time $t \in [0, T]$. We normalize all seven components in the SIDAREV model. In other words, we set the entire population equal to 1, as:

$$S(t) + I(t) + D(t) + A(t) + R(t) + E(t) + V(t) = 1$$

We will use the ANNs to obtain the optimal solution of the presented model, so here we give brief information about ANN.

Applying the PMP and the ANN

Consider the optimal control problem as follows:

$$\begin{aligned}
&\text{Min} \int_{t_0}^{t_f} f_0(x(t), u(t), t)\,dt,\\
&\dot{x} = g(x(t), u(t), t),\\
&x(t_0) = x_0,
\end{aligned} \tag{3}$$

where $x(t) \in R^n$ and $u(t) \in R^m$ are the state and control variables, respectively, and $t \in R$. It is assumed that g is Lipschitz continuous on a set $\Omega \subset R^n$, and t_0 and t_f are fixed; also, the integrand f_0 has continuous first and second partial derivatives for all its arguments.

PMP is a tool to create an ODE system in which the state and co-state variables are satisfied in the optimal conditions.

Theorem 1: If (x, u) is an optimal solution of (3), then there exists a piecewise differentiable adjoint function $\lambda(t)$, such that

$$\frac{\partial H(x,u,t,\lambda)}{\partial x} = -\dot{\lambda}(t), \tag{4}$$

$$\frac{\partial H(x,u,t,\lambda)}{\partial \lambda} = \dot{x}(t), \tag{5}$$

$$\frac{\partial H(x,u,t,\lambda)}{\partial u(t)} = 0. \tag{6}$$

where H is the following Hamiltonian:

$$H(x(t), u(t), \lambda(t), t) = f_0(x(t), u(t), t) + \lambda(t).g(x(t), u(t), t).$$

The proof can be found in (37).

A system of ODEs is constructed by equations (4), (5), and (6) that can be solved by numerous numerical methods. To solve the obtained equations, we made an attempt to propose an approximation scheme.

We define the trial functions, so that the initial conditions are satisfied, so we have:

$$\begin{cases} x_T = x_0 + (t - t_0)n_x,\\ \lambda_T = (t - t_f)n_\lambda,\\ u_T = n_u. \end{cases} \tag{7}$$

It is clear that x_T satisfies the initial condition, $x_T(t_0) = x_0$. If $x(t_f)$ is free, then we must have $\lambda(t_f) = 0$. Note that n_x, n_u, and n_λ are the neural networks for the state, control, and co-state functions, respectively. Each ANN contains its particular adjustable parameters. The proposed ANN is in the following form:

$$\begin{cases} n_x = \sum_{i=1}^{n} v_x^i \sigma(w_x^i t + b_x^i),\\ n_\lambda = \sum_{i=1}^{n} v_\lambda^i \sigma(w_\lambda^i t + b_\lambda^i),\\ n_u = \sum_{i=1}^{n} v_u^i \sigma(w_u^i t + b_u^i), \end{cases}$$

where b is a vector containing bias weight, w is a weight vector of the input layer, v is a weight vector of the output layer, and σ is an arbitrary activation function. In this work, the activation function is $\sigma(x)=\frac{1}{1+e^{-x}}$. Based on Kolmogorov's theorem (38), we can implement any continuous function for an MLP. As an example, Figure 2 shows the overview of the neural network of n_x.

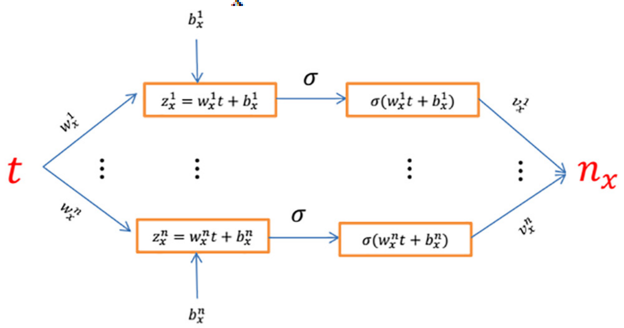

Figure 2: Overview of the neural network of n_x.

By replacing the trial solution in relations (4), (5), and (6), the following unconstrained optimization problem can be obtained.

$$\min E(\eta) = \frac{1}{2}\{E_1(t,\eta) + E_2(t,\eta) + E_3(t,\eta)\}, \qquad (8)$$

where $\eta = (w_x, w_\lambda, w_u, b_x, b_\lambda, b_u, v_x, v_\lambda, v_u)$, and

$$\begin{cases} E_1(t,\eta) = \left[\dfrac{\partial H_T}{\partial x_T} + \dot{\lambda}_T\right]^2, \\ E_2(t,\eta) = \left[\dfrac{\partial H_T}{\partial \lambda_T} - \dot{x}_T\right]^2, \\ E_3(t,\eta) = \left[\dfrac{\partial H_T}{\partial u_T}\right]^2, \end{cases} \qquad (9)$$

also,

$$H_T(x_T(t),u_T(t),\lambda_T(t),t) = f_{0_T}(x_T(t),u_T(t),t) + \lambda_T(t).g_T(x_T(t),u_T(t),t).$$

We discretize the interval $[t_0, t_f]$ by m points for solving the unconstrained optimization problem (8). To solve (8), an optimization algorithm such as Newton, steepest descent, or Quasi-Newton methods, etc., can be applied.

Now according to the explanations provided about the ANN, we examine the optimal control problem related to the SIDAREV model. First, the Hamiltonian function is constructed; then, PMP is used to find the conditions of the optimal control problem (39), and finally, the solution is given by ANNs. The Hamiltonian function can be defined as follows:

$$\begin{aligned} H(x,u,\lambda,t) = {} & \tfrac{c_1}{2}A(t)^2 + \tfrac{b_1}{2}u_1(t)^2 + \tfrac{b_2}{2}u_2(t)^2 + \tfrac{b_3}{2}u_3(t)^2 + \lambda_s\Big(-\beta S(t)I(t)(1-u_1(t)) - \\ & \psi S(t)u_3(t)\Big) + \lambda_I\Big(\beta S(t)I(t)(1-u_1(t)) - \gamma_i I(t) - \xi_i I(t) - \nu I(t)u_2(t)\Big) + \\ & \lambda_D(\nu I(t)u_2(t) - \gamma_d D(t) - \xi_d D(t) + \lambda_A\Big(\xi_i I(t) + \xi_d D(t) - \gamma_a A(t) - \bar{\mu}(A(t))\Big) + \\ & \lambda_R\big(\gamma_i I(t) + \gamma_d D(t) + \gamma_a A(t)\big) + \lambda_E\Big(\bar{\mu}(A(t))\Big) + \lambda_V\big(\psi S(t)u_3(t)\big), \end{aligned} \qquad (10)$$

where the values λ_S, λ_I, λ_D, λ_A, λ_R, λ_E, and λ_V are the associated adjoints for the state variables S, I, D, A, R, E, and V, respectively. Due to relation (5), the dynamic system of relation (1) is obtained. Based on relation (4), we create the adjoint system as:

$$\begin{aligned} \dot{\lambda}_S(t) &= -\frac{\partial H}{\partial s} = \lambda_S(t)\Big(\beta I(t)(1-u_1(t)) + \psi u_3(t)\Big) - \lambda_I(t)\beta I(t)(1-u_1(t)) - \lambda_V(t)\psi u_3(t), \\ \dot{\lambda}_I(t) &= -\frac{\partial H}{\partial I} = \lambda_S(t)\beta S(t)(1-u_1(t)) - \lambda_I\Big(\beta S(t)(1-u_1(t)) - \gamma_i - \xi_i - \nu u_2(t)\Big) - \lambda_D(t)\nu u_2(t) \\ &\quad - \lambda_A(t)\xi_i - \lambda_R(t)\gamma_i, \\ \dot{\lambda}_D(t) &= -\frac{\partial H}{\partial D} = \lambda_D(t)(\gamma_d + \xi_d) - \lambda_A(t)\xi_d - \lambda_R(t)\gamma_d, \\ \dot{\lambda}_A(t) &= -\frac{\partial H}{\partial A} = \lambda_A(t)\Big(\gamma_a + \bar{\mu}(A(t))\Big) - \lambda_E(t)\bar{\mu}(A(t)) - c_1 A(t) - \lambda_R(t)\gamma_a, \\ \dot{\lambda}_R(t) &= -\frac{\partial H}{\partial R} = 0, \\ \dot{\lambda}_E(t) &= -\frac{\partial H}{\partial E} = 0, \\ \dot{\lambda}_V(t) &= -\frac{\partial H}{\partial V} = 0. \end{aligned} \qquad (11)$$

Due to the relation (6), we have:

$$\frac{\partial H}{\partial u_1} = b_1 u_1(t) - \lambda_S(t)\beta S(t)I(t) + \lambda_I(t)\beta S(t)I(t) = 0,$$

$$\frac{\partial H}{\partial u_2} = b_2 u_2(t) - \lambda_I(t)\nu I(t) + \lambda_D(t)\nu I(t) = 0,$$

$$\frac{\partial H}{\partial u_3} = b_3 u_3(t) - \lambda_S(t)\psi S(t) + \lambda_V(t)\psi S(t) = 0,$$

based on the optimality conditions, we conclude:

$$\begin{aligned} u_1^*(t) &= \min\left[0.8, \max\left(0, \frac{(\lambda_I(t) - \lambda_S(t))\beta S(t)I(t)}{b_1}\right)\right], \\ u_2^*(t) &= \min\left[1, \max\left(0, \frac{(\lambda_I(t) - \lambda_D(t))\nu I(t)}{b_2}\right)\right], \\ u_3^*(t) &= \min\left[1, \max\left(0, \frac{(\lambda_S(t) - \lambda_V(t))\psi S(t)}{b_3}\right)\right]. \end{aligned} \qquad (12)$$

Now, we will construct the trial solution for state and co-state functions using the MLP. Note that because S, I, D, A, R, E, and V do not have fixed values at the final time, the values of the associated adjoints at the final time are zero. The trial solution of the state and co-state functions is defined as:

$$\Theta_T = \Theta_0 + t\left(\sum_{i=1}^{n} v_\Theta^i * \sigma(w_\Theta^i t + b_\Theta^i)\right),$$

and

$$\Lambda_T = (t-T)\left(\sum_{i=1}^{n} v_\Lambda^i * \sigma(w_\Lambda^i t + b_\Lambda^i)\right),$$

where Θ is S, I, D, A, R, E, and V variables, and Λ is λ_S, λ_I, λ_D, λ_A, λ_R, λ_E, and λ_V variables. It is necessary to remember that state and co-state variables have their weights and biases vectors.

The basis of solving the SIDAREV model

by ANN is as follows: first, we set the initial value of the control variables equal to zero ($u_i(t) = 0;\ 0 \le t \le T, i = 1,2,3$). Then, by replacing the trial solutions in the relation (1) and (11), the first and second of relations (9) are made. Finally, after obtaining new values from them, according to relations (12), the control variables are updated. We repeat this process until the distance of all the components are very close to the components of the previous step. In the MATLAB program related to this issue, we have set this distance equal to 10^{-6}.

Numerical Results

This section describes the experiments related to optimizing control inputs u_1, u_2, and u_3. ANNs are applied to consider the SIDAREV model. In this work, a left point Reimann sum is incorporated into the programs to approximate the objective functional. The neural network related to each of the variables includes fifteen adjustable weights, five weights of which are considered for each of the input layer, output layer, and bias. By placing the trial solutions in the first and second parts of relations (9), all these weights are updated by the neural network; then, controllers are updated by relation (12). The proposed scheme was tested in two experiments. The optimal control problem with the values is shown in Tables 1 and 2. The testing rates are taken from the research of Kasis et al. in (18). Assume that 0.0001% of the population is infected with the virus and no detected, acutely symptomatic, deceased, recovered, or vaccinated individuals at the start of the pandemic.

Since $S + I + D + A + R + E + V = 1$, the susceptible population is $1 - 0.0001\%$. The pandemic is simulated on $[0, T]$, where T equals 365 days.

In this part, we apply ANN for solving the SIDAREV model. It is clear that state variables can be formulated as follows:

$$S_T(t) = 1 - .00001 + tn_S,$$

$$I_T(t) = .00001 + tn_I,$$

$$D_T(t) = tn_D,$$

$$A_T(t) = tn_A,$$

$$R_T(t) = tn_R,$$

$$E_T(t) = tn_E,$$

$$V_T(t) = tn_V.$$

Also, for the associated adjoint variables, we have:

$$\lambda_{S_T}(t) = (t - T)n_{\lambda_S},$$

$$\lambda_{I_T}(t) = (t - T)n_{\lambda_I},$$

Table 1: Values of parameters

Value	Description
$\beta = 2/3$	Infection rate susceptible individuals
$\gamma_i = 1/14$	Recovery rate undetected individuals
$\gamma_i = 1/14$	Recovery rate detected individuals
$\gamma_i = 1/12.4$	Recovery rate threatened individuals
$\nu = 0 - 0.10$	Rate of detection of infected individuals (level of testing)
$\xi_i = 0.0053$	Rate infected individuals threatened
$\xi_d = 0.0053$	Rate infected detected individuals threatened
$h = (00333)$	Healthcare capacity
$\mu = 0.0085$	Mortality rate of disease
$\bar{\mu} = 5\mu$	Mortality rate of disease when healthcare capacity is exceeded
$\psi = 0, 0.0025$	Vaccination rate of susceptible individuals

Table 2: Initial conditions of SIDAREV model

Value	Description
$S_0 = 1 - .00001$	initial susceptible population
$I_0 = .00001$	initial infected undetected population
$D_0 = 0$	initial infected detected population
$A_0 = 0$	initial acutely symptomatic population
$R_0 = 0$	initial recovered population
$E_0 = 0$	initial extinct population
$V_0 = 0$	initial vaccinated population

$$\lambda_{D_T}(t) = (t - T)n_{\lambda_D},$$

$$\lambda_{A_T}(t) = (t - T)n_{\lambda_A},$$

$$\lambda_{R_T}(t) = \text{constant},$$

$$\lambda_{E_T}(t) = \text{constant},$$

$$\lambda_{V_T}(t) = \text{constant}.$$

All the calculations were executed and implemented with MATLAB software in a system with 4 GB of RAM.

Experiment 1: In this experiment, no control inputs were optimized. It was assumed that no government interventions were carried out, no tests were taken, and no vaccinations were given. This means that all control inputs were set to 0. This experiment shows what the effects of a disease outbreak are when a disease can run its free course.

Almost the entire population is in the susceptible state at the start of the disease outbreak. It is clear that no infected detected individuals (D) can be seen in Figure 3, because testing was not done. In Figure 3, the approximate solutions of the state variables of the suggested idea are shown. The objective functional value was 0.

Experiment 2: In this experiment, the control inputs $u_1, u_2,$ and u_3 were optimized simultaneously. Based on a changing weight factor c_1, the influence of the control inputs can be analyzed. It was assumed that the weight factor was $c_1 = 50000$. Also, we had the following assumptions:

- The maximum testing rate ν is set to 0.1.
- The maximum vaccination rate ψ is set to 2.5/1000.
- The weight factors for b are assumed to be b_1 = 3, b_2 = 2 and b_3 = 1.

In Figure 4, some of the states and controls variables are displayed and the approximate solutions of the states and control variables of the suggested idea are compared with the Runge-Kutta method of the fourth order presented in (40). The objective functional value of the Runge-Kutta method was 66.7719, whereas the idea presented was 66.7204 (Table 3).

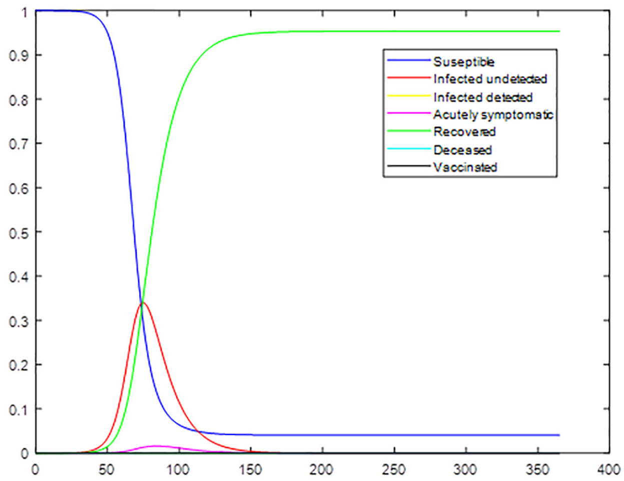

Figure 3: SIDAREV model without optimizing the control inputs

Discussion

In general, by considering all the conditions affecting the disease, it is possible to design a mathematical model that simulates the behavior of the disease. By applying controllers such as vaccination to the disease, its performance can be investigated using mathematical models. In this article, artificial neural network methods were used to show the effect of the controllers on the disease. By obtaining the Hamiltonian function related to the optimal

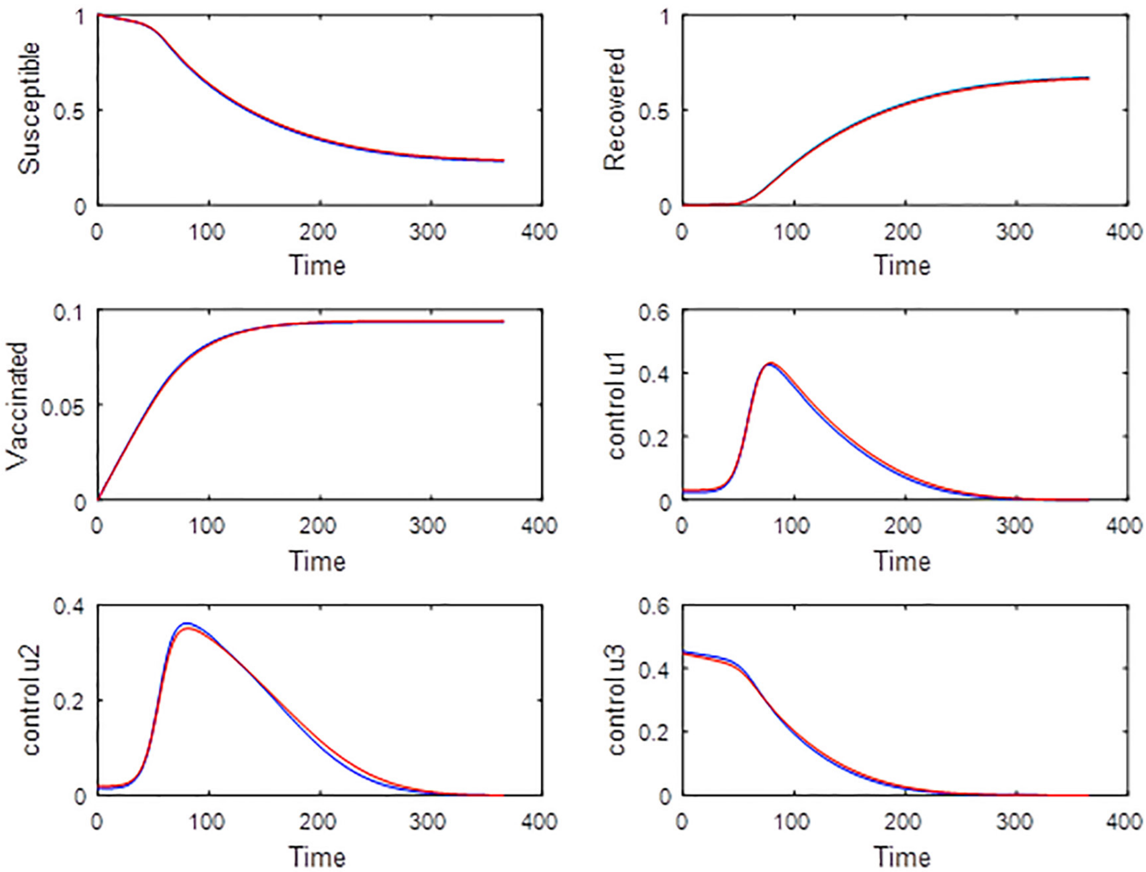

Figure 4: Optimization states and controls functions $c_1 = 50000$

Table 3: Values of functions related to experiment 2

Time		2	32	81	101	145	187	305	350
Susceptible	ANN	1	0.96	0.74	0.63	0.47	0.37	0.25	0.24
	Runge-Kutta	1	0.96	0.73	0.62	0.46	0.36	0.24	0.23
Recovered	ANN	0	0	0.12	0.22	0.39	0.50	0.64	0.66
	Runge-Kutta	0	0	0.13	0.23	0.40	0.51	0.65	0.67
Vaccination	ANN	0	0.034	0.073	0.082	0.090	0.093	0.094	0.094
	Runge-Kutta	0	0.034	0.075	0.083	0.091	0.093	0.094	0.094
Control u_1	ANN	0.032	0.040	0.431	0.367	0.209	0.106	0.004	0
	Runge-Kutta	0.024	0.032	0.423	0.353	0.195	0.093	0.002	0
Control u_2	ANN	0.019	0.033	0.351	0.329	0.240	0.143	0.006	0
	Runge-Kutta	0.014	0.028	0.361	0.337	0.236	0.131	0.003	0
Control u_3	ANN	0.445	0.423	0.272	0.201	0.092	0.037	0.001	0
	Runge-Kutta	0.452	0.434	0.267	0.190	0.082	0.030	0	0

control problem and applying the optimality conditions related to Pontriagin's minimum principle, we were able to form error functions for the state and co-state variables and put them in the dominant of an unconstrained optimality problem. Then, by combining the artificial neural network method with the forward-backward sweep method, the optimal values of the state, co-state and control variables were obtained. Since the type of error functions is convex, it can be concluded that in any situation, the results of this method are optimal and closer to the logical value. The related MATLAB software codes are generated, and used to simulate the diagrams related to the effects of controllers. Another advantage of this method is that the problem can be solved at a few limited points, but its value can be obtained at all points located on the interval. Comparing the presented method with Runge-Kutta method, we came to the conclusion that in some cases this method is more practical and useful.

Conclusion

By attributing the SIDAREV model to the COVID-19 disease, the behaviors of this disease can be analyzed both with and without a controller. By implementing policies such as government intervention, testing, and vaccination, we observed the behavior of the COVID-19 disease and came to the conclusion that this disease could be controlled to some extent. This happens when it can be investigated using mathematical models by presenting a model that simulates the behavior of the disease. In this work, inspired by the PMP for the SIDAREV model, the unconstrained optimization problem was designed. Then, we approximated the solutions of state and co-state variables of this problem using the ANN. With this, we observed the behavior of the disease considering different controllers over time. The most exciting characteristic of the ANNs is their capability to formulate problems using training. After sufficient training, the ANNs can solve problems of the same class since training algorithms converge to the optimal solutions.

To prove the capability of the ANN, it can be compared with the Runge-Kutta method. For future works, the proposed method can be utilized to solve other pandemic and epidemic models and fractional optimal control problems.

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Authors' Contribution

MD helped perform data extraction and selection. AY took charge of evaluating and compiling the contents. RHD analyzed the data, generated the MATLAB software code for the proposed approach, and prepared the manuscript. GA supervised the research, helped to generate MATLAB code, and edited the article. All authors reviewed the results and read and approved the article.

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Ethics Approval and Consent to Participate

This article is approved by the Ithenticate system for matching and the Grammarly system for language grammar.

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